Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 1 (Group)

香港数学竞赛 (2013 / 2014)

决赛项目 1 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若一个等腰三角形对应底边（不是两条等腰边）的高是 8，且周长是 32，求该三角形的面积。
If an isosceles triangle has height 8 from the base, not the legs, and perimeters 32, determine the area of the triangle.

2. 若 $f(x)=\dfrac{\left(x+\dfrac{1}{x}\right)^6-\left(x^6+\dfrac{1}{x^6}\right)-2}{\left(x+\dfrac{1}{x}\right)^3+\left(x^3+\dfrac{1}{x^3}\right)}$ 当中 x 是一个正实数，求 $f(x)$ 的最小值。
If $f(x)=\dfrac{\left(x+\dfrac{1}{x}\right)^6-\left(x^6+\dfrac{1}{x^6}\right)-2}{\left(x+\dfrac{1}{x}\right)^3+\left(x^3+\dfrac{1}{x^3}\right)}$ where x is a positive real number, determine the minimum value of $f(x)$.

3. 求 81 位数 $\overline{111\ldots1}$ 除以 81 的余数。
Determine the remainder of the 81-digit integer $\overline{111\ldots1}$ divided by 81.

4. 给定一实数数列 $a_1, a_2, a_3, \ldots$，它满足
1) $a_1=\dfrac{1}{2}$，及
2) 对 $k\geq 2$，有 $a_1+a_2+\cdots+a_k=k^2a_k$ 。
求 a_{100} 的值。
Given a sequence of real numbers $a_1, a_2, a_3, \ldots$ that satisfy
1) $a_1=\dfrac{1}{2}$, and
2) $a_1+a_2+\cdots+a_k=k^2a_k$, for $k\geq 2$.
Determine the value of a_{100} .

Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 2 (Group)

香港数学竞赛 (2013 / 2014)

决赛项目 2 (团体)

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Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若在 $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$ 中删去若干项后剩 1，求删去各项的乘积。

   By removing certain terms from the sum, $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{10}+\frac{1}{12}$, we can get 1. What is the product of the removed term(s)?

2. 若 $S_n=1-2+3-4+\cdots+(-1)^{n-1}n$，当中 n 是正整数，求 $S_{17}+S_{33}+S_{50}$ 的值。

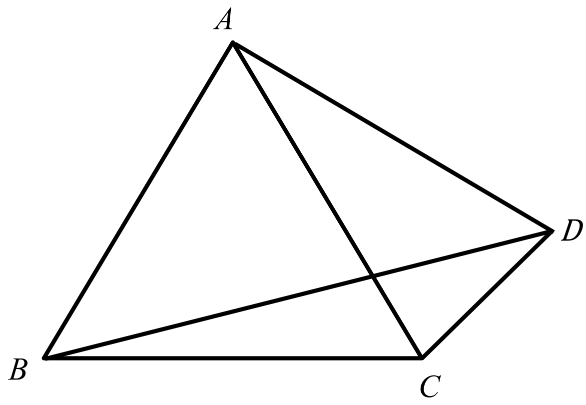
   If $S_n=1-2+3-4+\cdots+(-1)^{n-1}n$, where n is a positive integer, determine the value of $S_{17}+S_{33}+S_{50}$.

3. *A*, *B*, *C*, *D*, *E* 和 *F* 六人根据英文字母的顺序轮班工作。*A* 在第一个星期日当值，然后 *B* 在星期一当值，如此类推。*A* 于第 50 个星期的哪一天当值？（答案以数字 0 代表星期日，数字 1 代表星期一，……，数字 6 代表星期六）。

   Six people *A*, *B*, *C*, *D*, *E* and *F* are to rotate for night shifts in alphabetical order with *A* serving on the first Sunday, *B* on the first Monday and so on. In the fiftieth week, Which day does *A* serve on? (Represent Sunday by 0, Monday by 1, …, Saturday by 6 in your answer.)

4. 在下图中，D 以直线连接着等边三角形 ABC 的顶点，当中 $AB = AD$。设 $\angle BDC = \alpha°$，求 α 的值。

In the figure below, vertices of equilateral triangle ABC are connected to D in straight line segments with $AB = AD$. If $\angle BDC = \alpha°$, determine the value of α.



Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 3 (Group)

香港数学竞赛 (2013 / 2014)

决赛项目 3 (团体)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 求乘积 $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{10^2}\right)$ 的值。

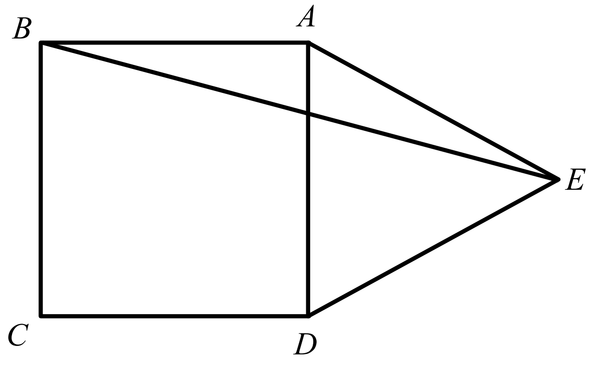
Determine the value of the product $\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{10^2}\right)$.

2. 求和 $\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\cdots+\frac{1}{\log_{100} 100!}$ 的值，当中 $100!=100\times 99\times 98\times\ldots\times 3\times 2\times 1$。

Determine the value of the sum $\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\cdots+\frac{1}{\log_{100} 100!}$, where $100!=100\times 99\times 98\times\ldots\times 3\times 2\times 1$.

3. 在下图中，*ABCD* 是一个正方形，*ADE* 是一个等边三角形，且 *E* 是正方形 *ABCD* 外的一点。设 $\angle AEB=\alpha°$，求 α 的值。

In the figure below, *ABCD* is a square, *ADE* is an equilateral triangle and *E* is a point outside of the square *ABCD*. If $\angle AEB=\alpha°$, determine the value of α.



4. 把不同的非零个位数填进下表白色的正方格内，使所有横、直的等式均成立。求 α。

Fill the white squares in the figure below with distinct non-zero digits so that the arithmetical expressions, read both horizontally and vertically, are correct. What is α?

	$\div$		$=$	
$+$		$\times$		
	$+$		$=$	α
$=$		$=$		

Hong Kong Mathematics Olympiad (2013 / 2014)

Final Event 4 (Group)

香港数学竞赛 (2013 / 2014)

决赛项目 4 (团体)

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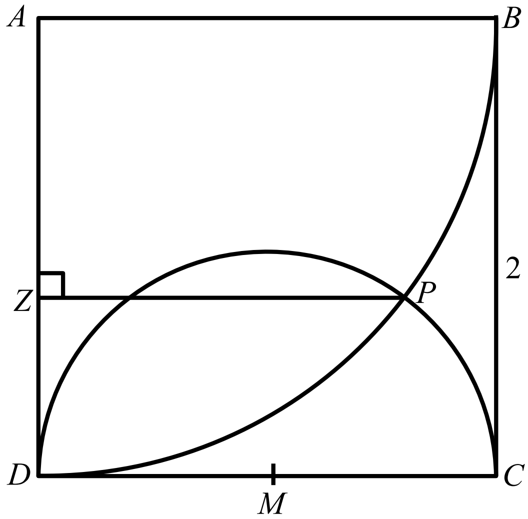
1. 在下图中，$ABCD$ 是一个边长为 2 的正方形。先以 A 为圆心画出弧 BD，再以 CD 的中点 M 为圆心从 C 到 D 画出一个半圆。弧 BD 和弧 DC 相交于 P。求 P 与 AD 的最短距离，即 PZ 的长度。

   In the figure below, $ABCD$ is a square of side length 2. A circular arc with centre at A is drawn from B to D. A semicircle with centre at M, the midpoint of CD, is drawn from C to D and sits inside the square. Determine the shortest distance from P, the intersection of the two arcs, to side AD, that is, the length of PZ.



2. 若 $x=\dfrac{\sqrt{5}+1}{2}$ 及 $y=\dfrac{\sqrt{5}-1}{2}$，求 $x^3y+2x^2y^2+xy^3$ 的值。

   If $x=\dfrac{\sqrt{5}+1}{2}$ and $y=\dfrac{\sqrt{5}-1}{2}$, determine the value of $x^3y+2x^2y^2+xy^3$.

3. 若 a, b, c 及 d 是不同的个位数，且

$$\begin{array}{r} a\ a\ b\ c\ d \\ -\ d\ a\ a\ b\ c \\ \hline 2\ 0\ 1\ 4\ d \end{array}$$

求 d 的值。

If a, b, c and d are distinct digits and

$$\begin{array}{r} a\ a\ b\ c\ d \\ -\ d\ a\ a\ b\ c \\ \hline 2\ 0\ 1\ 4\ d \end{array},$$

determine the value of d.

4. 求方程 $x^4+(x-4)^4=32$ 所有实根的乘积。

Determine the product of all real roots of the equation $x^4+(x-4)^4=32$.